Eur. Phys. J. C 51, 697–699 (2007)
DOI 10.1140/epjc/s10052-007-0320-9

THE EUROPEAN
PHYSICAL JOURNAL C

Regular Article – Theoretical Physics

An SU(3) symmetry for light neutrinos

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Received: 23 March 2007 / Revised version: 17 April 2007 /
Published online: 6 June 2007 –

Abstract. It is proposed that light neutrinos form a triplet in a global SU(3) symmetry in the mass eigenstate basis. Assuming that the SU(3) symmetry is broken in the direction $\left(-a\lambda_3+\frac{b}{\sqrt{3}}\lambda_8\right)$, and after going to the flavor basis, we predict the atmospheric mixing angles $\sin^2\theta_{23}=0.5$ and $\sin\theta_{13}=0$, if ν_μ–ν_τ symmetry is assumed. In the flavor basis, the diagonal part of the matrix coefficient of b (the dominant part) is found to transform like $\left(\lambda_3+\frac{1}{\sqrt{3}}\lambda_8\right)$. Imposing the same condition on the matrix coefficient of a fixes the solar mixing angle, $\sin^2\theta_{12}=\frac{1}{3}$. The implications for neutrinoless double beta decay are discussed.

There is compelling evidence (for a recent discussion and many references, see [1]) that neutrinos change flavor, have non-zero masses and that the neutrino mass eigenstates are different from the weak eigenstates. As such they undergo oscillations.

All neutrino data [1] with the exception of the LSND anomaly [2] are explained by three neutrino flavor oscillations with mass squared differences and mixing angles having the following values [3]:

$$\begin{aligned}\Delta m^2_{\rm solar} &= \Delta m^2_{12} = (8.1\pm1.0)\times10^{-5}\,{\rm eV}^2\,,\\ \sin^2\theta_{12} &= 0.30\pm0.08\,,\\ \Delta m^2_{\rm atm} &= \left|\Delta m^2_{13}\right| \simeq \left|\Delta m^2_{23}\right|\\ &= (2.2\pm1.1)\times10^{-3}\,{\rm eV}^2\,,\\ \sin^2\theta_{23} &= 0.50\pm0.18\,,\\ \sin^2\theta_{13} &\le 0.047\,. \end{aligned} \tag{1}$$

The above mixing pattern is in conformity with the "bi-tri-maximal" scheme, first discussed in [4, 5]. The neutrino mixing angles are defined by the lepton mixing matrix [6]

$$\begin{pmatrix}\nu_e\\ \nu_\mu\\ \nu_\tau\end{pmatrix} = U\begin{pmatrix}\nu_1\\ \nu_2\\ \nu_3\end{pmatrix}\,. \tag{2}$$

The matrix U is conveniently parametrized by the three mixing angles θ_{12}, θ_{13} and θ_{23} and the three complex phases, two of which are the so-called Majorana phases. We put all phases to zero.

The purpose of this paper is to study the implications of SU(3) symmetry for light neutrinos in the mass eigenstate basis. SU(3) family symmetry has previously been used [7, 8] in a more fundamental way than attempted here. Our aim is modest. We show that if the SU(3) symmetry for the neutrino mass eigenstates is broken in the direction $(-a\lambda_3+\frac{b}{\sqrt{3}}\lambda_8)$, and if we go to the flavor basis by the unitary transformation given in (2), the atmospheric mixing angles are predicted to be $\sin^2\theta_{23}=0.5$ and $\sin\theta_{13}=0$, if the ν_μ–ν_τ symmetry is assumed. For recent use of $\nu_e\longleftrightarrow\nu_\mu$ symmetry in other contexts, see for example, [9–11]. This symmetry is inspired by the experimental observation of a near maximal atmospheric mixing angle and a small upper limit on θ_{13}, implying the interesting possibility that there may be an approximate $\mu\leftrightarrow\tau$ symmetry in the neutrino sector [12–22]. Its deeper origin is not yet known. Such a symmetry has also interesting implications in leptogenesis; see, for example, [23–25].

It is found that in a flavor basis, the diagonal part of the matrix coefficient of b (dominant part) exhibits an interesting property, namely that it transforms like $\left(\lambda_3+\frac{1}{\sqrt{3}}\lambda_8\right)$. If we impose the same condition on the matrix coefficient of a (non-leading part) we also predict the solar mixing angle $\sin^2\theta_{13}=\frac{1}{3}$. In our approach the absolute mass of the neutrino in the SU(3) limit, m_0, is not constrained by neutrino oscillation data. If the WMAP constraint on the neutrino mass, $\sum m_i<(0.4$–$0.7)\,{\rm eV}$, is used, the effective double beta decay mass $m_{ee}<(0.13$–$0.23)\,{\rm eV}$.

We assume that the mass eigenstates (ν_1,ν_2,ν_3) form an SU(3) triplet, so that, in the SU(3) limit, they have the common mass m_0. Since the neutrino mass matrix in the mass eigenstate basis has to be diagonal and the only diagonal matrices available are λ_3 and λ_8, the most general form of symmetry breaking is provided by $(-a\lambda_3+\frac{b}{\sqrt{3}}\lambda_8)$,

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so that in the basis (ν_1, ν_2, ν_3) we have

$$\mathcal{M} = m_0 I + \left(-a\lambda_3 + \frac{b}{\sqrt{3}}\lambda_8\right), \tag{3}$$

$$\begin{aligned} m_1 &= m_0 - a + \frac{b}{3} = m - a, \\ m_2 &= m_0 + a + \frac{b}{3} = m + a, \\ m_3 &= m_0 - \frac{2b}{3} = m - b, \end{aligned} \tag{4}$$

where $m = m_0 + \frac{b}{3}$ and $|a|$, $|b| << m_0$.

Thus, we have

$$\begin{aligned} \Delta m_{12}^2 &\simeq 4ma, \\ \left|\Delta m_{23}^2\right| &\simeq 2m(a+b) + (b^2 - a^2), \\ \left|\Delta m_{13}^2\right| &\simeq 2m(b-a) + (b^2 + a^2). \end{aligned} \tag{5}$$

The data require that $a \ll b$, so that $m_3 \ll m_1 \preceq m_2$ (inverted hierarchy) and

$$\left|\Delta m_{23}^2\right| \simeq \left|\Delta m_{13}^2\right| \simeq 2mb. \tag{6}$$

We now go to the flavor basis $(\nu_e, \nu_\mu, \nu_\tau)$ by using (2), $[s_1 = \sin\theta_{23},\ c_1 = \cos\theta_{23}$ and $s_2 \equiv \sin\theta_{13}]$. The neutrino mass matrix in the flavor basis then is

$$M_\nu = m_0 I + M,$$

with

$$\begin{aligned} M_{11} &= \frac{b}{3}\left(1 - 3s_2^2\right) - ac_2^2\cos 2\theta_{12}, \\ M_{22} &= \frac{b}{3}\left(1 - 3s_1^2 c_2^2\right) \\ &\quad - a\left[\cos 2\theta_{12}\left(-c_1^2 + s_1^2 s_2^2\right) + \sin 2\theta_1 \sin 2\theta_{12} s_2\right], \\ M_{33} &= \frac{b}{3}\left(1 - 3c_1^2 c_2^2\right) \\ &\quad - a\left[\cos 2\theta_{12}\left(-s_1^2 + c_1^2 s_2^2\right) - \sin 2\theta_1 \sin 2\theta_{12} s_2\right], \\ M_{12} &= -bc_2 s_2 s_1 + ac_2\left[c_1 \sin 2\theta_{12} + s_1 s_2 \cos 2\theta_{12}\right], \\ M_{13} &= -bc_2 s_2 c_1 - ac_2\left[s_1 \sin 2\theta_{12} - c_1 s_2 \cos 2\theta_{12}\right], \\ M_{23} &= -bc_1 s_1 c_2^2 \\ &\quad - a\left[s_1 c_1 \cos 2\theta_{12}\left(1 + s_2^2\right) - \sin 2\theta_{12} s_2\left(s_1^2 - c_1^2\right)\right]. \end{aligned} \tag{7}$$

Imposing $\nu_\mu \longleftrightarrow \nu_\tau$ symmetry, we get

$$s_2 = 0, \quad c_1 = -s_1 = \frac{1}{\sqrt{2}}.$$

Thus, M reduces to

$$\frac{b}{2}\begin{pmatrix} \frac{2}{3} & 0 & 0 \\ 0 & -\frac{1}{3} & 1 \\ 0 & 1 & -\frac{1}{3} \end{pmatrix} - \frac{a}{2}\begin{pmatrix} 2\cos 2\theta_{12} & -\sqrt{2}\sin 2\theta_{12} & -\sqrt{2}\sin 2\theta_{12} \\ -\sqrt{2}\sin 2\theta_{12} & -\cos 2\theta_{12} & -\cos 2\theta_{12} \\ -\sqrt{2}\sin 2\theta_{12} & -\cos 2\theta_{12} & -\cos 2\theta_{12} \end{pmatrix}. \tag{8}$$

It is interesting to note that the diagonal part of the matrix coefficient of $\frac{b}{2}$ transforms as $\lambda_3 + \frac{1}{\sqrt{3}}\lambda_8$, like the electric charge operator of the u, d and s quarks. If we require the same for the matrix coefficient of $-a/2$, we obtain

$$\cos 2\theta_{12} = \frac{1}{3},$$

giving

$$\sin^2\theta_{12} = \frac{1}{3}.$$

This is consistent with its experimental value given in (1). Thus, the neutrino mass matrix in the flavor basis is

$$M_\nu = m_0 I + \frac{b}{2}\begin{pmatrix} 2/3 & 0 & 0 \\ 0 & -1/3 & 1 \\ 0 & 1 & -1/3 \end{pmatrix} - \frac{a}{2}\begin{pmatrix} 2/3 & -4/3 & -4/3 \\ -4/3 & -1/3 & -1/3 \\ -4/3 & -1/3 & -1/3 \end{pmatrix}. \tag{9}$$

The data give

$$\begin{aligned} \frac{a}{b} &\simeq \frac{1}{2}\frac{\Delta m_{12}^2}{\left|\Delta m_{23}^2\right|} \simeq 1.8 \times 10^{-2} \\ \frac{b}{m_0} &\simeq \frac{1}{2}\frac{\left|\Delta m_{23}^2\right|}{m_0^2} \simeq (1.1) \times 10^{-3}\,\frac{\text{eV}^2}{m_0^2}. \end{aligned} \tag{10}$$

m_0 is not constrained by the oscillation data. However, m_0 is constrained by the WMAP data, $\sum m_i < (0.4–0.7)$ eV.

Thus, taking

$$m_0 \simeq 0.1\ \text{eV},$$

we have

$$\frac{b}{m_0} \approx 10^{-1}. \tag{11}$$

The SU(3) symmetry thus makes sense, as the symmetry breaking parameter is small.

Finally, for neutrinoless double β decay, the effective double beta decay mass $\langle m_{ee}\rangle$ is predicted to be [σ_1 and σ_2 are Majorana phases, $\theta_{13} = 0$] [26]

$$\begin{aligned} m_{ee} &= \left||m_1|\cos^2\theta_{12}\mathrm{e}^{-2\mathrm{i}\sigma_1} + |m_2|\sin^2\theta_{12}\mathrm{e}^{-2\mathrm{i}\sigma_2}\right| \\ &\simeq \frac{m_0}{3} < m_{ee} < m_0. \end{aligned} \tag{12}$$

Using the WMAP limit on m_0, we get

$$m_{ee} \leq (0.13–0.23)\ \text{eV}.$$

In summary, a global SU(3) symmetry for the neutrino mass eigenstates and its breaking along the direction $(-a\lambda_3 + \frac{b}{\sqrt{3}}\lambda_8)$ with $a \ll b$, together with the $(\nu_\mu \longleftrightarrow \nu_\tau)$ symmetry in the flavor basis and the requirement that the diagonal part of the neutrino mass matrix $(M_\nu - m_0 I)$ transforms as $\left(\lambda_3 + \frac{1}{\sqrt{3}}\lambda_8\right)$, can explain the data in (1).

Furthermore, together with the WMAP constraint on the neutrino mass, an effective double beta decay mass m_{ee} is predicted.

Acknowledgements. The author acknowledges a research grant provided by the Higher Education Commission of Pakistan to him as a Distinguished National Professor.

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